Measures of Efficiency in the Environmental Context

Misure di efficienza in campo ambientale

Filippa Bono
Università degli Studi di Palermo
Dipartimento di Contabilità Nazionale ed Analisi dei Processi Sociali
e-mail: bono@economia.unipa.it

Domenica Matranga
Università degli Studi di Palermo
Dipartimento di Biotecnologie Mediche e Medicina Legale
Sezione Scienze Radiologiche
e-mail:domenicamatranga@unipa.it

Riassunto: Il concetto di efficienza ambientale è di estrema importanza per i policy-makers nel raggiungimento dello sviluppo sostenibile. Inoltre, la conoscenza del proprio "livello di eco-efficienza" può essere importante nell'attività di programmazione per il progresso di un Paese. In questo lavoro suggeriamo un metodo, da noi denominato "context-dependent eco-efficiency DEA" che, basandosi sulla metodologia DEA, permette di ottenere una misurazione dell'efficienza ambientale delle unità analizzate, anche in presenza di output indesiderabili. Il ricorso a tale modello permette, nel contempo, di valutare i livelli di efficienza e di tenere conto del contesto in cui l'unità opera e che la caratterizza. Il metodo è qui impiegato per analizzare l'eco-efficienza dipendente dal contesto delle regioni europee sulla base di indicatori ambientali raccolti dal data set Eurostat.

Keywords: DEA, undesirable outputs, eco-efficiency.

1. Introduction

The question of eco-efficiency is of great importance for policy-makers, as a consequence of the priority to achieve sustainable development of human societies. When we want to measure the concept of ecological efficiency we must take into account undesirable outputs. Given that context is very important to determine the eco-efficiency of a unit, we introduce a context dependent eco-efficiency DEA model. To policy makers, the knowledge of their own "level of eco-efficiency" can be crucial and in need of improvement.

The structure of the paper is the following: in the next section we present a review of DEA models with undesirable output and we introduce the context-dependent eco-efficiency DEA. The suggested methods are applied in section 3 to data from Eurostat database on environment and energy. Conclusions and references follow.

2. DEA models with undesirable outputs

Let us consider *n* decision making units (DMUs) which are responsible of converting *m* inputs into *p* outputs and whose performances are to be evaluated. Data Envelopment Analysis (DEA) model in its original formulation suggested by Charnes, Cooper and Rhodes (Charnes et al., 1978) is a non-parametric method to measure the efficiency of each DMU with respect to the others as a ratio of a weighted sum of outputs to a weighted sum of inputs subject to the condition that the ratio of outputs to inputs should not exceed 1 for each DMU.

In classical DEA only desirable outputs are used. To consider eco-efficiency we must take into account undesirable outputs, like pollutants for instance, which need to be minimized. There are several ways to incorporate undesirable outputs within the model (Korhonen et al. 2004). The primal presentations of these models allow a unified presentation called General Model CCR (**C**harnes, **C**ooper and **R**hodes) primal-dual.

Let be $\mathbf{Y}^g \in \Re_+^{d\times n}$, $\mathbf{Y}^b \in \Re_+^{(p-d)\times n}$, $\mathbf{X} \in \Re_+^{m\times n}$ the matrices of the *d* desirable outputs, the (*p*-*d*) undesirable outputs and the *m* inputs. Let $\mathbf{y}^g, \boldsymbol{\mu}^g$ and $\mathbf{s}^g$ vectors in $\Re_+^d$; $\mathbf{y}^b, \boldsymbol{\mu}^b$ and $\mathbf{s}^b$ vectors in $\Re_+^{p-d}$ and $\mathbf{v}, \mathbf{x}$ and $\mathbf{s}^-$ vectors in $\Re_+^m$. Finally, we denote $\mathbf{1} = [1,1,...1]^T$. We have the following primal-dual LP model pair:

General model CCR primal:

$$
\begin{aligned}
&\max \quad g_G = \sigma + \varepsilon \mathbf{1}^T\left(\mathbf{s}^b + \mathbf{s}^g + \mathbf{s}^-\right) \\
&\textit{subject to} \quad \mathbf{Y}^g\boldsymbol{\lambda} - \boldsymbol{\sigma}\mathbf{w}^g - \mathbf{s}^g = \mathbf{y}_o^g \\
&\qquad\qquad\quad\;\; \mathbf{Y}^b\boldsymbol{\lambda} + \boldsymbol{\sigma}\mathbf{w}^b + \mathbf{s}^b = \mathbf{y}_o^b \\
&\qquad\qquad\quad\;\; \mathbf{X}\boldsymbol{\lambda} + \boldsymbol{\sigma}\mathbf{w}^x + \mathbf{s}^- = x_o \\
&\lambda, s^-, s^g, s^b \geq 0 \qquad \varepsilon > 0
\end{aligned}
$$

General model CCR dual: (1)

$$
\begin{aligned}
&\min \quad h_G = -\boldsymbol{\mu}_g^T\mathbf{y}_o^g + \boldsymbol{\mu}_b^T\mathbf{y}_o^b + \mathbf{v}^T\mathbf{x}_o \\
&\textit{subject to} \quad \boldsymbol{\mu}_g^T\mathbf{w}^g + \boldsymbol{\mu}_b^T\mathbf{w}^b + \mathbf{v}^T\mathbf{w}^x = 1 \\
&\qquad\qquad\quad\;\; -\boldsymbol{\mu}_g^T\mathbf{Y}^g + \boldsymbol{\mu}_b^T\mathbf{Y}^b + \mathbf{v}^T\mathbf{X} \geq \mathbf{0} \\
&\boldsymbol{\mu}_g, \boldsymbol{\mu}_b, \mathbf{v} \geq \mathbf{0}
\end{aligned}
$$

The optimal solution of the CCR dual model is given by the vectors of weights $\boldsymbol{\mu}_g, \boldsymbol{\mu}_b, \mathbf{v}$ which minimize the mix of input and undesirable output, while maximizing the desirable output, of the DMU_o being evaluated. There are several ways to specify undesirable outputs. It is demonstrated that the efficient units are efficient in all model variants, even if the efficiency scores may differ (see Korhonen et al. 2004). In the following, we refer to DEA method with variants for undesirable inputs with the term eco-efficiency DEA model. In particular, we choose that model variant which treats undesirable outputs as inputs and corresponds to $\mathbf{w}^g = \mathbf{w}^b = \mathbf{0}$ and $\mathbf{w}^x = \mathbf{x}_o$ and $\sigma = 1 - \theta$.

Afterwards, we incorporate eco-efficiency DEA model within a context-dependent framework. Context-dependent DEA evaluates a set of DMUs against a particular evaluation context which is given by an efficient frontier composed by DMUs in a specific performance level. Let us consider a set of *n* DMUs. If we remove the original efficient frontier, then the remaining inefficient DMUs will form a new second level efficient frontier. In a similar way, if we remove this new second level efficient frontier, a third level efficient frontier is constituted and so on until no more DMUs remain (Seiford, Zhu, 2003).

We propose the following context-dependent eco-efficiency DEA:

Step 1: Set $r = 1$. Perform DEA analysis to the entire data-set D_1 using the primal-dual LP model pair and choosing one of DEA model variants for undesirable outputs. Let us denote the first-level eco-efficient frontier $EE^{(1)}$. For $1 \le r \le n$, we have:

$$EE^{(r)} = \left\{ DMU \in D_r \mid \left(\sigma^* = 0\right) \cap \left(s^{b*} = 0, s^{g*} = 0, s^{-*} = 0\right)\right\}.$$

Step 2: Exclude the *r*-level eco-efficient units form the whole data-set in order to obtain a new smaller data-set $D_{r+1} = D_r - EE^{(r)}$. Perform the eco-efficiency DEA model on the reduced data-set D_{r+1} and let $EE^{(r+1)}$ be the *(r+1)*-level eco-efficient frontier.

Step 3: Set $r = r + 1$. If $D_r \neq \varnothing$ go back to *step 2*, else the algorithm stops.

In order to evaluate the context associated to each level of eco-efficiency, we perform a principal component analysis of variables observed at each level. Let M be the $n \times (m+p)$ data matrix (assume $n \geq m+p$). If the number of cases contained in $EE^{(r+1)}$ is less than the number of input and output considered, we aggregate $EE^{(r+1)}$ to the previous level $EE^{(r)}$.

3. An application to European countries

The method has been tested on data collected from Eurostat database relative to environment. Variables considered in the analysis are *total greenhouse gas emissions* (I1), *energy intensity of the economy* (I2), *transport* (I3), *municipal waste collected* (I4), *share of renewable energy* (I5) and *investments to environmental protection* (I6).

Since undesirable outputs can be treated as inputs, in this application we considered only one input (I6). Because of several missing data found in the last indicator only 17 European countries have been considered. The algorithm's output consists in two levels of eco-efficiency named $EE^{(1)}$ and $EE^{(2)}$ (Table 1):

Table 1: *Levels of eco-efficiency*

$EE^{(1)}$	$EE^{(2)}$
Finland – Sweden – Latvia – Norway – Poland – Austria – Denmark – Germany	France – Greece – Belgium – Italy – Luxemburg United Kingdom – Netherlands – Ireland – Spain

On each group we performed PCA and obtained two factors respectively (Table 2). The cumulative proportion of variance explained by two factors in $EE^{(1)}$ together is 88%. In $EE^{(2)}$, two factors together explain the 75% of total variance. We can notice that factor loadings associated to $EE^{(2)}$ differ from those associated to $EE^{(1)}$ only for indicator I5.

Table 2: *Levels of eco-efficiency*

Indicators	$EE^{(1)}$		$EE^{(2)}$	
	I factor	**II factor**	**I factor**	**II factor**
I1	**-0,927**	-0,117	**-0,821**	0,523
I2	**0,979**	0,057	0,148	**0,770**
I3	-0,009	**0,969**	**-0,900**	0,333
I4	**-0,842**	0,306	-0,420	**-0,860**
I5	-0,434	**0,768**	-0,304	0,382
I6	**-0,783**	-0,557	**0,779**	0,474
% Explained variance	55.5%	32.5%	39.7%	35.3%

Therefore, the percentage of renewable energy is the only indicator which discriminates between the two levels of eco-efficiency. The policy maker interested to improve his own level of eco-efficiency could think convenient to advance toward increasing levels of this indicator.

4. Conclusions

In this paper we introduced a new approach to deal with undesirable outputs which we named context dependent eco-efficiency DEA. We measured the context dependent eco-efficiency DEA of the European countries on the base of Eurostat dataset on environment. The analysis is performed in two steps. First, we estimated a context dependent DEA which consists of two different strata of efficient frontiers rather than the traditional unique-level efficient frontier used in traditional approach. Second, on each strata of efficient European countries we performed a PCA and obtained two factors respectively. The analysis of factor loadings revealed that the percentage of renewable energy is the only indicator which discriminates between the two levels of eco-efficiency.

5. References

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